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Applications of Mohand Transform for Solving Linear Volterra Integral Equations of First Kind

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Abstract-In this paper, we deals with the Mohand transform for solving the linear Volterra Integral equations of the first kind. Some applications are given in according to reveal the effectiveness of solving the linear Volterra Integral equations of the first kind using Mohand transform. The Mohand transform, inverse Mohand transform and the convolution theorem are used in this study to derive the exact solution.

Keywords-Linear Volterra integral equations of first kind; Mohand transform; Inverse Mohand transform

1. INTRODUCTION

An Integral equations arise in many scientific and engineering problems. In mathematics the volterra integral equations are the special type of integral equations. The linear volterra integral equation of first kind is given by [1-4]

$$f(x) = \int_0^x k(x,t)\,u(t)\,dt \_\_\_\_\_\_\_\_ (1)$$

where the unknown function $u(x)$, that will be determined, occurs only inside the integral sign. The kernel $k(x,t)$ and the function $f(x)$ are given in the real valued functions.

Mohand transform is derived from the classical Fourier integral. The Mohand transform of the function $f(t)$ is defined as [9].

$$M\left[f(t)\right] = R(v) = v^2 \int_0^\infty f(t)e^{-vt}dt,\ t \ge 0,\ k_1 \le v \le k_2$$

where M is a Mohand operator. The Mohand transform of the function $f(t)$ for $t \ge 0$ exist if $f(t)$ is piecewise continuous and of exponential order. These conditions are only sufficient for the existence of Mohand transform of the function $f(t)$.

Aggarwal et al. [5] discussed a new application of Kamal Transform for solving linear Volterra integral equations. Also Aggarwal et al. [6] discussed a new application of Mahgoub transform for solving linear Volterra integral equations. Haarsa [7] solved on Volterra integral equations of the first kind by using Elzaki transform. Aggarwal et al. [8] discussed an application of Aboodh transforms for solving linear Volterra integro – differential equations of second kind.

The aim of this paper work is to establish exact solutions for linear Volterra integral equations of first kind using Mohand Transform without any difficulty.

2. MOHAND TRANSFORM FOR LINEAR VOLTERRA INTEGRAL EQUATIONS OF FIRST KIND

In this work we will assume that the kernel $k(x,t)$ "Eq. (1)" is a difference $(x-t)$, the linear Volterra integral equation of first kind "Eq.(1)" can be expressed as

$$f(x) = \int_0^x k(x-t)\,u(t)\,dt \_\_\_\_\_\_\_ (2)$$

3. MOHAND TRANSFORM OF SOME STANDARD RESULTS

S.No.	f(t)	M{f(t)}=R(v)
1.	1	v
2.	t	1
3.	t^n	$\frac{n!}{v^{n-1}}$
4.	e^{at}	$\frac{v^2}{v-a}$
5.	$\sin at$	$\frac{av^2}{v^2+a^2}$
6.	$\cos at$	$\frac{v^3}{v^2+a^2}$

7.	$\sinh at$	$\frac{av^2}{v^2-a^2}$
8.	$\cosh at$	$\frac{v^3}{v^2-a^2}$

4. INVERSE MOHAND TRANSFORM OF STANDARD RESULTS

S.No.	R(v)	f(t)=M^{-1}{R(v)}
1.	v	1
2.	1	t
3.	$\frac{n!}{v^{n-1}}$	t^n
4.	$\frac{v^2}{v-a}$	e^{at}
5.	$\frac{av^2}{v^2+a^2}$	$\sin at$
6.	$\frac{v^3}{v^2+a^2}$	$\cos at$
7.	$\frac{av^2}{v^2-a^2}$	$\sinh at$
8.	$\frac{v^3}{v^2-a^2}$	$\cosh at$

5. CONVOLUTION OF MOHAND TRANSFORM

Convolution of two functions $F(t)$ and $G(t)$ is denoted by $F(t)*G(t)$ and it is defined by

$$F(t)*G(t)=F*G=\int_0^t F(x)G(t-x)dx=\int_0^t F(t-x)G(x)dx$$

$$M\left[f(t)*g(t)\right]=\frac{1}{2}M\left[f(t)\right]\square M\left[g(t)\right]$$

6. APPLICATIONS

In this section, some applications are given in order to demonstrate the effectiveness of Mohand transform for solving linear Volterra integral equations of first kind.

6.1. ***Application 1*** Consider the equation

$$x=\int_0^x e^{(x-t)}u(t)dt \text{ \_\_\_\_\_\_\_\_(3).}$$

Applying the Mohand transform on both sides of "Eq.(3)", we have

$$M(x)=M\left[\int_0^x e^{(x-t)}u(t)dt\right]$$

Using convolution theorem of Mohand transform and simplify, we have

$$M(x)=M\left[e^x*u(x)\right]$$
$$=\frac{1}{v^2}M\left[e^x\right]\bullet M\left[u(x)\right]$$
$$=v-1$$

Take Inverse Mohand transform on both sides, we have

$$u(x)=M^{-1}(v)-M^{-1}(1)=1-x \text{ \_\_\_\_\_\_\_\_(4)}$$

Which is the required exact solution of "Eq.(3)".

6.2. ***Application 2*** Consider the equation

$$\sin x=\int_0^x e^{(x-t)}u(t)dt \text{ \_\_\_\_\_\_\_\_(5).}$$

Applying the Mohand transform on both sides of "Eq.(5)", we have

$$M(\sin x)=M\left[\int_0^x e^{(x-t)}u(t)dt\right] \text{\_\_\_\_\_\_\_\_(6)}$$

Using convolution theorem of Mohand transform and simplify, we have

$$\frac{v^2}{v^2+1}=M\left[e^x*u(x)\right]$$
$$=\frac{1}{v^2}\frac{v^2}{v-1}\bullet M\left[u(x)\right]$$
$$M\left[u(x)\right]=\frac{v^2(v-1)}{v^2+1}=\frac{v^3}{v^2+1}-\frac{v^2}{v^2+1}$$

Taking Inverse Mohand transform, we have

$$u(x)=M^{-1}\left(\frac{v^3}{v^2+1}\right)-M^{-1}\left(\frac{v^2}{v^2+1}\right)$$
$$=\cos x-\sin x \text{ \_\_\_(7)}$$

Which is the required exact solution of "Eq.(5)".

6.3. ***Application 3*** Consider the equation

$$x^2=\frac{1}{2}\int_0^x (x-t)u(t)dt \text{ \_\_\_\_\_\_\_\_(8).}$$

Applying the Mohand transform on both sides of "Eq.(8)", we have

$$M\left(x^2\right)=\frac{1}{2}M\left[\int_0^x (x-t)u(t)dt\right] \text{\_\_\_\_\_\_\_\_\_\_(9)}$$

Using convolution theorem of Mohand transform and simplify, we have

$$\frac{2}{v}=\frac{1}{2}M\left[x*u(x)\right]$$
$$=\frac{1}{2}\frac{1}{v^2}M(x)\bullet M\left[u(x)\right]$$
$$=\frac{1}{2v^2}M\left[u(x)\right]$$
$$M\left[u(x)\right]=4v$$

Taking inverse Mohand transform on both sides, we have

$$u(x)=4M^{-1}(v)=4\_\_\_\_\_\_\_\_\_\_(10)$$

Which is the required exact solution of “Eq.(8)”.

6.4. ***Application 4*** Consider the equation

$$x=\int_0^x e^{-(x-t)}u(t)\,dt\_\_\_\_\_\_\_\_\_\_(11).$$

Applying the Mohand transform on both sides of “Eq.(11)”, we have

$$M(x)=M\left[\int_0^x e^{-(x-t)}u(t)\,dt\right]\_\_\_\_\_\_\_\_\_\_(12)$$

By convolution theorem, we have

$$1=\frac{1}{v^2}M\left(e^{-x}\right)\square M\left[u(x)\right]$$
$$=\frac{1}{v^2}\left(\frac{v^2}{v+1}\right)M\left[u(x)\right]$$
$$v+1=M\left[u(x)\right]$$

Taking inverse Mohand transform, we have

$$u(x)=M^{-1}[v]+M^{-1}[1]\_\_\_\_\_\_\_\_(13)$$
$$=1+x$$

Which is the required exact solution of “Eq.(11)”.

6.5. ***Application 5*** Consider the equation

$$x=\int_0^x u(t)\,dt\_\_\_\_\_\_\_\_\_\_(14).$$

Applying the Mohand transform on both sides of “Eq.(14)”, we have

$$M(x)=M\left[\int_0^x u(t)\,dt\right]\_\_\_\_\_\_\_\_(15)$$

By convolution theorem, we have

$$1=\frac{1}{v^2}M(1)\square M\left[u(x)\right]$$
$$=\frac{1}{v^2}v\square M\left[u(x)\right]$$
$$v=M\left[u(x)\right]$$

Taking inverse Mohand transform, we have

$$u(x)=M^{-1}[v]=1\_\_\_\_\_\_\_\_\_\_(16)$$

Which is the required exact solution of “Eq.(14)”.

7. CONCLUSION

In this paper, we have developed the Mohand transform for solving linear Volterra integral equations of first kind. The applications are solved by using Mohand transform showing the exact solution. The calculation part takes not much of difficulty, but also taking little time. Like other integral transforms, this Mohand transform is also for solving volterra integral equations of first kind easily. The methodology can be also applied for other linear Volterra Integral Equations and for their system.

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